EE 118 MT 1 reference sheet.

LECTURE 1

Newton: light is a particle
Huygen: light is a ray
Einstein: light is BOTH

wave functions

$f(x,t)$ - waves do not have to be periodic
- waves do NOT have to be sinusoidal

Travelling waves: the argument of a travelling wave must follow the format $\boxed{ax+bt}$ with velocity $=\boxed{-\frac{b}{a}}$

//Note: speed $=\left|\frac{b}{a}\right|$, $\boxed{\text{if } a \,\&\, b \text{ have same sign} \rightarrow -x \text{ dir.} \quad \text{if } a \,\&\, b \text{ have diff. signs} \rightarrow +x \text{ dir.}}$

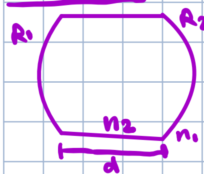
Huygen's Principle: every point on a wavefront can be treated as a point source for secondary wavelets

Refractive index $\boxed{n=\frac{c}{v}}$ //speed of light in vacuum / // speed of light in medium

what causes dispersion?: varying indices of refraction by λ

Lecture 4

Thick lenses.



$$\begin{bmatrix}x_o\\ \theta_o\end{bmatrix}=\underbrace{\begin{bmatrix}1 & 0\\ \frac{n_2-n_1}{R_2 n_1} & \frac{n_2}{n_1}\end{bmatrix}}_{\text{curve 2}}\underbrace{\begin{bmatrix}1 & d\\ 0 & 1\end{bmatrix}}_{\text{propagate}}\underbrace{\begin{bmatrix}1 & 0\\ \frac{n_1-n_2}{R_1 n_2} & \frac{n_1}{n_2}\end{bmatrix}}_{\text{curve 1}}\begin{bmatrix}x_i\\ \theta_i\end{bmatrix}$$

Thin-lens approx:

$$\underline{M}=\begin{bmatrix}1 & 0\\ \frac{n_2-n_1}{R_2 n_1} & \frac{n_2}{n_1}\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{n_1-n_2}{R_1 n_2} & \frac{n_1}{n_2}\end{bmatrix}=\boxed{\begin{bmatrix}1 & 0\\ -\frac{1}{f} & 1\end{bmatrix}}$$

Lensmaker's Eq: $\boxed{\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$ (also lens POWER)

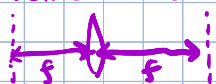
//evidently, lens power is related to refraction index difference

consecutive lenses: $\begin{bmatrix}1 & 0\\ -\frac{1}{f_1} & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -\frac{1}{f_2} & 1\end{bmatrix}=\boxed{\begin{bmatrix}1 & 0\\ -\frac{1}{f_1}-\frac{1}{f_2} & 1\end{bmatrix}}$

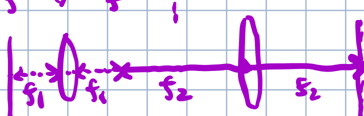
thus $\boxed{f_{cascade}=\left(-\frac{1}{f_1}-\frac{1}{f_2}\right)^{-1}}$ & $\boxed{\Phi_{cascade}=\Phi_1+\Phi_2}$

Hack: determinant of an ABCD matrix is the ratio of refractive indices n_1/n_2.

2f system: $\begin{bmatrix}0 & f\\ -1/f & 0\end{bmatrix}$



4f system: $\begin{bmatrix}-f_1/f_2 & 0\\ 0 & -f_2/f_1\end{bmatrix}$



determinants of both $=1$: since $n_{in}=n_{out}$

Maxwell's imaging conditions ① all rays from point converge to point ② Images of all points ⊥ to lens axis lie on a plane ⊥ to axis ③ Ratio obj h/img h const.

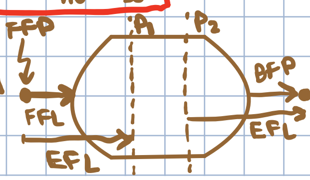
Imaging condition (from B=0): $\boxed{\frac{1}{S_o}+\frac{1}{S_i}=\frac{1}{f}}$

Lecture 5

$\boxed{x_o x_i=f^2}$

$\boxed{M=\frac{-h_i}{h_o}=\frac{-S_i}{S_o}}$

real img ⟷ inverted
virtual img ⟷ erect.
principal planes →



$\boxed{NA=n\sin\theta}$ ← This is the smallest θ allowed in system by the aperature stop.

$\boxed{f\#=1/2(NA)}$

$\boxed{EFL=FFL+PP_1}$
$\boxed{EFL=BFL+PP_2}$

Lecture 2

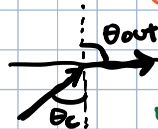
3 ways light can travel: → ① through empty space ② through some medium ③ reflecting off surface

Fermat's theorem: light from point A → B will always seek the SHORTEST OPL. $\boxed{OPL=ns=\Sigma n_i s_i}$

Transit time $=\frac{S_1}{v_1}+\frac{S_2}{v_2}+\dots\frac{S_x}{v_x}=\boxed{\Sigma S_i/v_i=\frac{1}{c}\Sigma n_i s_i=\frac{OPL}{c}}$

Snell's Law: $\boxed{n_1\sin\theta_1=n_2\sin\theta_2}$ //proof via lifeguard problem.

Critical Angle: $n_1\sin\theta_c=n_2\sin(90°)\rightarrow 1$



$\theta_{out}=90°\Rightarrow\boxed{\theta_c=\sin^{-1}(n_2/n_1)}$

Lecture 3

How is a ray defined?

$\Rightarrow\begin{bmatrix}y_{in}\\ \theta_{in}\end{bmatrix}$ { Paraxial optics (matrix method ok) $\theta\approx\tan\theta\approx\sin\theta$, $\cos\theta\approx 1$

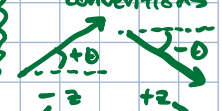
Ray propagation:
$x_{out}=x_{in}+d\tan\theta_{in}$
$\theta_{out}=\theta_{in}$ paraxial approx $\Rightarrow\boxed{\begin{bmatrix}1 & d\\ 0 & 1\end{bmatrix}}$

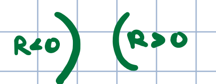
Ray refractions:
$x_{out}=x_{in}$
$n_{in}\sin\theta_{in}=n_{out}\sin\theta_{out}$ → $\theta_{out}=\frac{n_{in}}{n_{out}}\theta_{in}$ (paraxial approx) $\boxed{\begin{bmatrix}1 & 0\\ 0 & \frac{n_{in}}{n_{out}}\end{bmatrix}}$

General form of ABCD matrix:

$$\begin{bmatrix}x_{out}\\ \theta_{out}\end{bmatrix}=\begin{bmatrix}\partial x_{out}/\partial x_{in} & \partial x_{out}/\partial\theta_{in}\\ \partial\theta_{out}/\partial x_{in} & \partial\theta_{out}/\partial\theta_{in}\end{bmatrix}\begin{bmatrix}x_{in}\\ \theta_{in}\end{bmatrix}$$

Common sign conventions

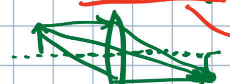




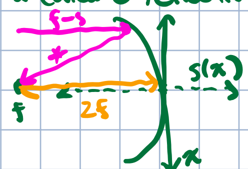
For FFL $\partial\theta_{out}/\partial\theta_{in}=\boxed{D=0}$

For BFL $\partial x_{out}/\partial x_{in}=\boxed{A=0}$

For Imaging $\frac{\partial x_{out}}{\partial\theta_{in}}=0$ $\boxed{B=0}$



What is the optimal geometry for a concave reflecting mirror?



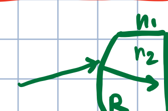
$\boxed{2f=f-s+\sqrt{x^2+(f-s)^2}}$

//essentially OPL should match

$\boxed{s(x)=x^2/4f}$

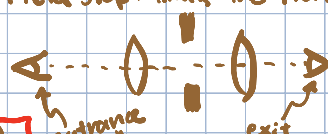
Ray over spherical surface:

$$\begin{bmatrix}x_{out}\\ \theta_{out}\end{bmatrix}=\boxed{\begin{bmatrix}1 & 0\\ \frac{n_1-n_2}{Rn_2} & \frac{n_1}{n_2}\end{bmatrix}}\begin{bmatrix}x_{in}\\ \theta_{in}\end{bmatrix}$$



sanity check: as $R\rightarrow\infty$, the ABCD matrix matches Snell's law

Aperature stop: limits Quantity of light in/out
Field stop: limits the F.O.V. (an angle)

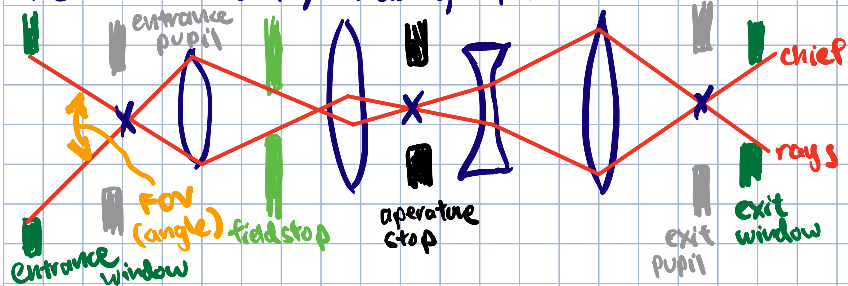


entrance pupil, exit pupil. → images of AS as seen from before or after lens system.

$\boxed{FOV=2\tan^{-1}\left(\frac{h}{2f}\right)}$ where h = size of sensor, f = focal length of lens

marginal: on the edge
vs
chief: through center of AS

LECTURE 6 | A clearer representation of all the different stops, windows, & pupils:



- note: all chief rays pass through the CENTERS of the entr. P, exit P, and AS.
- all chief rays pass through the EDGES of FS, entrance window, & exit window.
- "windows" are images of field stops before/after system
- "pupils" are images of aperature stops before/after

EYE TRIVIA
- Farsighted: (hypermetropity) converging
- nearsighted (myopia) diverging lens

optical devices:
- Microscope: Zacharius Janssen 1590,
- compound microscope: Galileo 1610
- telescope: Janssen OR Hans Lippershy 1608
- gravitational lensing: using gravity as light lens!

Lecture 8 | image plane: 1-D pupil-plane: 2-D

$$\boxed{\epsilon_x = \frac{1}{n'\theta_a'}\frac{\partial w}{\partial \rho_x} \qquad \epsilon_y = \frac{1}{n'\theta_a'}\frac{\partial w}{\partial \rho_y}}$$

ϵ_x, ϵ_y: how far off the real ray is from paraxial (ideal) as a function of (h, ρ, ϕ)

$n' \rightarrow$ medium index

W → wavefront shape

$\theta_a \rightarrow$ angle of marginal ray

$\rho \rightarrow$ pupil coord

Wavefront aberration function: $\boxed{W(h,\rho,\phi) = W_r(h,\rho,\phi) - W_p(h,\rho,\phi)}$ // ideally $= 0$!

invariant forms of h, ρ, ϕ under rotation: $h^2, \rho^2, h\rho\cos\phi$

∴ General form $\boxed{W(h,\rho,\phi) = \sum_{ijk} W_{ijk} h^i \rho^j \cos^k\phi}$

// all terms in summation must be invariant,
// $i + j = 0, 2, 4, 6 \ldots$

Example of 4th-order wavefront aberration fct.

$$\boxed{W(h,\rho,\phi) = W_{040}\rho^4 + W_{131} h\rho^2[\rho\cos\phi] + W_{220} h^2\rho^2 + W_{222} h^2\rho^2\cos^2(\phi) + W_{311} h^3\rho\cos(\phi) + W_{400} h^4}$$

// spherical + coma + Petzval + astigmatism + distortion + piston

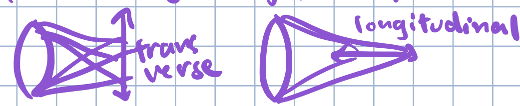
LECTURE 7 | Aberrations in imaging

Abbe number $\boxed{V = \frac{f_D^{-1}}{f_F^{-1} - f_C^{-1}} = \frac{n_D - 1}{n_F - n_C}}$

↓ Abbe ⟺ ↑ dispersion

where D ≙ 588 nm (roughly yellow light)
F ≙ 486 nm (blue light)
C ≙ 656 nm (red light)

Transverse vs longitudinal aberrations



Chromatic, (1st order) Achromatic, (2nd order) $f_F = f_C$ Apochromat. (3rd order) $f_F = f_C = f_D$!

Achromatic doublet

$$\frac{1}{f} = \boxed{\frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}} = (n_1 - 1)R_1 + (n_2 - 1)R_2 - d(n_1-1)R_1(n_2-1)R_2$$

// d is distance btw lenses.

when $d = 0$: $\frac{1}{f_R} = \frac{1}{f_B}$ ✓, $\frac{R_1}{R_2} = -\frac{n_{2B} - n_{2R}}{n_{1B} - n_{1R}}$

@ any 2 wavelengths: $\boxed{f_{1y}V_1 + f_{2y}V_2 = 0}$

Minimizing Chromatic Aberrations

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2}$ using lensmakers

$\frac{1}{f_1} = (n-1)\left(\frac{1}{R_{11}} - \frac{1}{R_{12}}\right) = C_1(n-1)$

$\frac{1}{f_2} = (n-1)\left(\frac{1}{R_{21}} - \frac{1}{R_{22}}\right) = C_2(n-1)$

minimization step: $\frac{d(1/f)}{dn} = C_1 + C_2 - 2LC_1C_2(n-1) = 0$

$\boxed{L = \frac{1}{2}(f_1 + f_2)}$ // to reduce chromatic ab, make dist between lenses = avg of lens focal lengths

Third-order approximation $\sin\theta \approx \theta - \theta^3/3!$

Types of aberrations

- chromatic
- monochromatic
  - blurring
    - coma, spherical, astigmatism
  - distortion
    - Field curvature, distortion

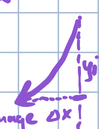
Spherical | - causes longitudinal aberr. - use an asphere instead
- looks like concentric blurry rings.

Coma | - causes transverse aberr. - looks like a 'tailed' image
- applies to off-axis rays

Astigmatism | - vertical & horz light have different foci
- there is a vertical focus, horz focus, and compromise

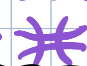
Field curvature | - image is on a curved-surface.
- use curved sensor

$\boxed{\Delta x = \frac{y_i^2}{2}\sum_{j=1}^{m}\frac{1}{n_j f_j}}$ // amount of curved displacement



Distortion | Barrel →

Pincushion →



EE118 MT2 Cheatsheet

Maxwell's Equations

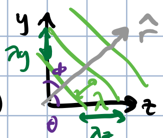
Math Background

$\tilde{z} = x + iy$
$= A(\cos\varphi + i\sin\varphi)$
$= Ae^{i\varphi}$ ⟷ Euler's Relation.

Phasor notation

$Ae^{i\varphi} + Ae^{-i\varphi} = 2A\cos(\varphi)$
$Ae^{i\varphi} + Ae^{i(-\varphi+\pi)} = 2iA\sin(\varphi)$
$E_0\cos(kz - \omega t) = E_0 e^{ikz}$
where $k = 2\pi/\lambda$

Plane waves:



$\vec{k} = \frac{2\pi}{\lambda}\hat{r} = \frac{2\pi}{\lambda_y}\hat{y} + \frac{2\pi}{\lambda_z}\hat{z}$

$\lambda_z = \frac{\lambda}{\cos\theta}$ $\quad \lambda_y = \frac{\lambda}{\cos\varphi}$

Drop temporal term in phasor

Real: $\Psi(\vec{r},t) = A\cos\left(\frac{2\pi}{\lambda}(\cos\varphi y + \cos\theta z) \cancel{+ \omega t}\right)$

Phasor: $\Psi(\vec{r}) = A\exp\left(i\frac{2\pi}{\lambda}(\cos\varphi y + \cos\theta z)\right)$

// $\cos\varphi y + \cos\theta z = \frac{y}{r}y + \frac{z}{r}z = \frac{y^2}{r} + \frac{z^2}{r} = \frac{r^2}{r} = r$

Spherical waves: $\boxed{\text{General Form: } \vec{E}(\vec{r},t) = \frac{E_0}{r}\cos(kr - \omega t)}$

- spherical waves become plane waves as $r \to \infty$

$$\boxed{\begin{aligned}&\text{General Form for plane waves:}\\ &E(\vec{r}) = E_0\cos\vec{k}\cdot\vec{r} = E_0\exp\{i\vec{k}\cdot\vec{r}\}\\ &= E_0 e^{i\frac{2\pi}{\lambda}r}\end{aligned}}$$

MAXWELL's EQs.

$\nabla \times \vec{E} = -\partial\vec{B}/\partial t$	Faraday's Law
$\nabla \times \vec{H} = \partial\vec{D}/\partial t + \vec{J}$	Ampere's Law.
$\nabla \cdot \vec{D} = \rho$	Coulomb's / Gauss' Law for E-fields
$\nabla \cdot \vec{B} = 0$	Gauss' Law.

VARIABLES

$\vec{E}$ [V/m] electric field
$\vec{B}$ [Wb/m²] magnetic flux density
$\vec{D}$ [C/m²] Electric displacement
$\vec{H}$ [Amp/m] Magnetic Field
$\vec{J}$ [Amp/m²] E-current density
ρ [C/m³] Charge density.

RELATIONS

$\vec{D} = \varepsilon\vec{E}$ $\quad \varepsilon = \varepsilon_0\varepsilon_r$ "permittivity" ("relative permittivity")
$\vec{B} = \mu\vec{H}$ $\quad \mu = \mu_0\mu_r$ "permeability"
$n = \sqrt{\mu_r\varepsilon_r}$ // material refractive ind
$c = (\mu_0\varepsilon_0)^{-1/2}$ // speed of light in vacuum

Poynting Vector $\vec{S}$
$\vec{S} = c^2\varepsilon_0\vec{E}\times\vec{B}$
// describes ray direction
// parallel to propagation
// $|\vec{S}| \propto$ Power/Area of surface

MAXWELL'S EQs INTEGRAL FORM.

$\oint_C \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\iint_A \vec{B}\cdot d\vec{a}$
$\oint_C \vec{H}\cdot d\vec{l} = \iint_A \vec{J}\cdot d\vec{a} + \iint_A \frac{\partial\vec{D}}{\partial t}\cdot d\vec{a}$
$\oiint_A \vec{D}\cdot d\vec{a} = \iiint_V \rho\, dV$
$\oiint_A \vec{B}\cdot d\vec{a} = 0$

Math Review for Maxwell's eqs.

Gradient: $\nabla E = \{\partial E/\partial x, \partial E/\partial y, \partial E/\partial z\}$
Divergence: $\nabla\cdot\vec{E} = \partial E_x/\partial x + \partial E_y/\partial y + \partial E_z/\partial z$
Curl: $\nabla\times\vec{E} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$

Stoke's Thm: $\int \vec{F}\cdot d\vec{r} = \oiint \text{curl}\,\vec{F}\cdot d\vec{S} = \iint \text{curl}\,\vec{F}\cdot\hat{n}\,dA$
Divergence Thm: $\oiint \vec{F}\cdot d\vec{S} = \iiint \text{div}(\vec{F})\cdot dV$

Physics 7B Review

$\vec{E} = q/(4\pi\varepsilon_0 r^2)\,\hat{r}$ // Proved via Gauss' law.
$\vec{B} = (\mu/4\pi)(I\,d\vec{l}\times\hat{r})/|\vec{r}|^2$
$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) = \underbrace{q\vec{E}}_{\text{electrostatic}} + \underbrace{q\vec{v}\times\vec{B}}_{\text{magnetic (Lorentz)}}$

POLARIZATION

General form of polarization: $\vec{E} = \vec{E}_x + \vec{E}_y$
$= \hat{x}E_{0x}\cos(kz - \omega t) + \hat{y}E_{0y}\cos(kz - \omega t)$

$\boxed{\text{Linear: } \vec{E} = (E_0\cos(\theta)\hat{x} + E_0\sin(\theta)\hat{y})\cos(kz - \omega t)}$

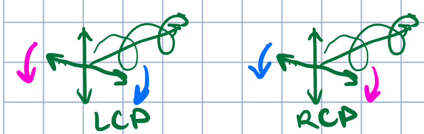
$\boxed{\text{Circular: } E = \hat{x}E_0\cos(kz - \omega t) + \hat{y}E_0\sin(kz - \omega t)}$

LCP vs RCP ($\mp$ respectively)

$\boxed{\vec{E} = \hat{x}E_0\cos(kz - \omega t) \mp \hat{y}E_0\sin(kz - \omega t)}$

Adding LCP + RCP, sin term cancels out.

$\vec{E}_{LCP} + \vec{E}_{RCP} = 2E_0[\hat{x}\cos(kz - \omega t)]$ // linearly polarized

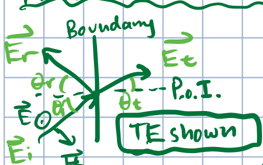


$\boxed{\text{Elliptical: } \vec{E} = \hat{x}E_0\cos(kz - \omega t) + \hat{y}E_1\sin(kz - \omega t)}$

// occurs when LCP & RCP do not perfectly cancel each other out.

Plane waves at boundaries

Parallel ⟺ S ⟷ TM
Perpendicular ⟺ P ⟷ TE
"relative to plane of incidence"



$\boxed{\vec{E}_i - \vec{E}_r = \vec{E}_t}$

Birefringence
varying refractive index → polarization

ordinary wave/axis: behaves as expected & $n = n_o$
extraordinary wave/axis: behaves differently & $n = n_e$

Uniaxial vs Biaxial Crystals

$\begin{bmatrix} n_o & 0 & 0 \\ 0 & n_o & 0 \\ 0 & 0 & n_e \end{bmatrix}$ vs $\begin{bmatrix} n_\alpha & 0 & 0 \\ 0 & n_\beta & 0 \\ 0 & 0 & n_\gamma \end{bmatrix}$

Malus' Law:
$\boxed{I_0 = I_i\cos^2\theta}$ $\quad I \propto A^2$
$A_0 = A_i\cos\theta$

Fresnel Equations & Brewster's angle

$r_s = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$ $\qquad r_p = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}$

$t_s = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2}$ $\qquad t_p = \frac{2n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}$

$\boxed{\theta_B = \arctan(n_2/n_1)}$ "angle where $r_p = 0$"

$\boxed{\eta = \frac{|E|}{|H|} = \sqrt{\frac{\mu}{\varepsilon}}}$ "impedance"

$\lambda/4$ & $\lambda/2$ wave plates

$OPD = d(|n_o - n_e|)$

Phase delay $= \Delta\varphi = \frac{2\pi}{\lambda} OPD$

$\boxed{\lambda/4 \text{ plate: } \varepsilon = \frac{\pi}{2} \quad d(|n_o - n_e|) = \frac{(4m+1)\lambda}{4}}$ Lin → Circ

$\boxed{\lambda/2 \text{ plate: } \varepsilon = \pi \quad d(|n_o - n_e|) = \frac{(2m+1)\lambda}{2}}$ Lin → 90° rotated Linear.

$\boxed{I_1 = \varepsilon_0 c \langle \vec{E}_1 \cdot \vec{E}_1 \rangle = \frac{1}{2}\varepsilon_0 c E_1^2}$ $\boxed{I_2 = \frac{1}{2}\varepsilon_0 c E_2^2}$ } substitute into

$\boxed{I_{12} = 2\sqrt{I_1 I_2}\langle \cos\Delta\varphi \rangle}$

Young's double-slit experiment.

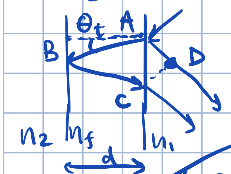
$\boxed{m\lambda = d\sin\theta}$

d: distance between slits θ: angle from midpoint of slits. $m = 0, 1, 2 \ldots$

paraxial approximation $\sin\theta \simeq \tan\theta = y/D \Rightarrow \boxed{y = \frac{m\lambda D}{d}}$

D: distance to imaging plane, y: location on img. plane

Thin-film interference



$\Delta OPL = \Lambda = n_f[(\overline{AB}) + (\overline{BC})] - n_1(\overline{AD})$

$\Lambda = 2 n_f d \cos\theta_t$ $\Delta\varphi = k\Lambda = \boxed{\delta = \frac{4\pi}{\lambda} n_f d\cos\theta_t \pm \pi}$

Multi-beam Interfer.

r, t: external reflection   r', t': internal reflection

d: thickness of film

Reflections:

$E_{1r} = r E_0 e^{i\omega t}$

$E_{2r} = t r' t' E_0 e^{i(\omega t - \delta)}$

$E_{3r} = t r'^3 t' E_0 e^{i(\omega t - 2\delta)}$

$E_{4r} = t r'^5 t' E_0 e^{i(\omega t - 3\delta)}$

Transmissions:

$E_{1t} = t t' E_0 \ldots$

$E_{2t} = t t' r'^2 E_0 \ldots$

$E_{3t} = t t' r'^4 E_0 \ldots$

$E_{4t} = t t' r'^6 E \ldots$

Stoke's Relations

$r' = -r$

$tt' = 1 - r^2$

Rule of thumb: r off higher n ⇒ π phase shift; r off lower n ⇒ no phase change

// infinite geometric sum $\left(\frac{a}{1-r}\right)$

$$\boxed{I_r = I_i \frac{2r^2(1-\cos\delta)}{1 + r^4 - 2r^2\cos\delta}} \quad \boxed{I_t = I_i \frac{(1-r^2)^2}{1 + r^4 - 2r^2\cos\delta}}$$

Interference

intensity as a measured time-averaged interference

$I = \varepsilon_0 c \langle \vec{E}\cdot\vec{E} \rangle$ interference from 2 sources

$\vec{E}_p = \vec{E}_1 + \vec{E}_2 \Rightarrow I_p = \varepsilon_0 c \langle \vec{E}_p \cdot \vec{E}_p \rangle$

$= \varepsilon_0 c \langle \underbrace{\vec{E}_1\cdot\vec{E}_1}_{I_1} + \underbrace{\vec{E}_2\cdot\vec{E}_2}_{I_2} + \underbrace{2\vec{E}_1\cdot\vec{E}_2}_{I_{12}} \rangle$ I_{12} interference term.

$\boxed{I_{12} = \varepsilon_0 c\, \vec{E}_{01}\cdot\vec{E}_{02} \langle \cos\Delta\varphi \rangle}$

where φ is the phase difference between the sources.

phase shifting interferometry

$I_{out} = |\vec{E}_{ref} + \vec{E}_{sig}|^2$

$= |A_r \exp(i\varphi_r) + A_s \exp(i\varphi_s)|^2$

phase-shifting interferometry creates 4 images

$I_{zero} = I_0[1 + m\cos(\varphi)]$ //what?

$I_n = I_0[1 + m\cos(\varphi + n)]$

Phase: $\tan\varphi = (I_{3\pi/2} - I_{\pi/2})/(I_0 - I_\pi)$

Amplitude: $A = \sqrt{\frac{I_0 + I_{\pi/2} + I_\pi + I_{3\pi/2}}{4}}$

Coherence

Spatial (Young's) & Temporal (Michelson's)

Requirements for interference: same λ, same polarization, COHERENCE,

incoherent: $I = I_1 + I_2$

coherent: $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle\cos\Delta\varphi\rangle$

Temporal Coherence functions:

$$\boxed{G_{12}(\tau) = \int_{-\infty}^{\infty} E_1(t)E_2(t+\tau)\,dt = \int_{-\infty}^{\infty} E_2(t)E_1(t-\tau)\,dt} \quad G_{12}(-\tau) = G_{12}^*(\tau)$$

$$0 < \left|\boxed{g_{12}(\tau) = \frac{G_{12}(\tau)}{\sqrt{G_{11}(0)G_{22}(0)}}} = \frac{G_{12}(\tau)}{\sqrt{I_1 I_2}}\right| < 1$$

denominator is the self-correlation

coherence time: $\tau_c = \int_{-\infty}^{\infty} |g(\tau)|^2 d\tau$

coherence length: $\ell_c = c\tau_c$

Power Spectral density: $S(\nu) = \int_{-\infty}^{\infty} G(\tau) e^{-i2\pi\nu\tau} d\tau$

Spatial Coherence area: $\boxed{A_c = D^2\lambda^2/\pi d^2}$

where D: distance   d: diameter of source

A_c: minimum aperture area needed to catch incoherence.

Diffraction:

Fourier transform: $g(x) = \int_{-\infty}^{\infty} \tilde{G}(f_x)\exp(i2\pi f_x x)\,df_x$

$\operatorname{sinc}(x) = \frac{\sin(x)}{x}$

Diffraction by a single slot of width d

governing integral by Huygen's principle: $E_\theta = \int_{-\frac{d}{2}}^{\frac{d}{2}} E_0 e^{ik(r - x\sin\theta)} dx$

$\Rightarrow I = |E_\theta|^2 = \boxed{E_0^2 d^2 \operatorname{sinc}^2\left(\frac{kd}{2}\sin\theta\right)}$ → dark spots @ $\sin\theta = \pm\frac{m\lambda}{d}$

Diffraction by double-slot with spacing a.

$\boxed{I = I_0 \operatorname{sinc}^2\left(\frac{d\pi\sin\theta}{\lambda}\right)\cos^2\left(\frac{a\pi\sin\theta}{\lambda}\right)}$ → bright @ $\pm\frac{n\lambda}{a}$ → dark @ $\pm\frac{m\lambda}{d}$

Optics as signal Processing

$g(x) = \int \underbrace{g(x')}_{\text{weights}} \underbrace{\delta(x - x')}_{\text{elementary functions}} dx'$

if we know system H's impulse response to $\delta(x)$: $h(x)$, then we know the response to $g(x)$

$\boxed{g_{out}(x) = \int g_{in}(x')\,h(x - x')\,dx'}$

convolution in real space ⇔ multiplication in Fourier space: $g(x)\otimes h(x) \xrightarrow{\mathcal{F}} \tilde{G}(\omega)\tilde{H}(\omega)$

Diffraction of circular aperature of diam d

first minima: $d = 1.22\frac{\lambda z}{w} \approx 1.22\lambda/\theta$

Fraunhofer Diffraction

$$E(x,y) = \frac{e^{ikz} e^{i k/2\pi (x^2+y^2)}}{j\lambda z} \iint_{-\infty}^{\infty} E(\xi,\eta)\exp\left[-j\frac{2\pi}{\lambda z}(x\xi + y\eta)\right] d\xi\,d\eta$$

at spatial frequencies $\boxed{f_x = x/\lambda z \quad f_y = y/\lambda z}$

$$\boxed{E(x,y) = \frac{e^{ikz} e^{i k/2\pi (x^2+y^2)}}{j\lambda z}\mathcal{F}\{E(\xi,\eta)\}}$$

Fresnel integral

field at obs. plane (x,y) from input plane (ξ,η)

$$E(x,y) = \frac{z}{j\lambda}\iint_\Sigma E(\xi,\eta)\frac{\exp(jkr_{01})}{r_{01}^2}\,d\xi\,d\eta$$

Σ → area in input plane

where $r_{01} = \sqrt{z^2 + (x-\xi)^2 + (y-\eta)^2}$

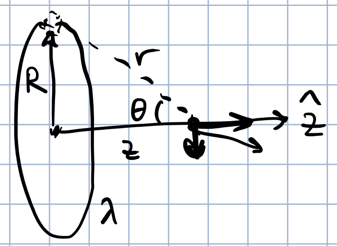
In the Fresnel regime $r_{01} \approx z\left[1 + \frac{1}{2}\left(\frac{x-\xi}{z}\right)^2 + \frac{1}{2}\left(\frac{y-\eta}{z}\right)^2\right]$

rewrite → $E(x,y) = \iint_{-\infty}^{\infty} E(\xi,\eta)\,h(x-\xi, y-\eta)\,d\xi\,d\eta$

$\boxed{h(x,y) = \frac{e^{jkz}}{j\lambda z}\exp\left[\frac{jk}{2z}(x^2+y^2)\right]}$ $\xrightarrow{\mathcal{F}}$

$\boxed{H(\omega_x,\omega_y) = e^{jkz}\exp[-j\pi\lambda z(\omega_x^2 + \omega_y^2)]}$

Quiz 2. practice



1a) $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$

$= \frac{q}{4\pi\varepsilon_0 (R^2+z^2)}(\cancel{\sin\theta\,\hat{x}} + \cos\theta\,\hat{z})$

x components will cancel out.

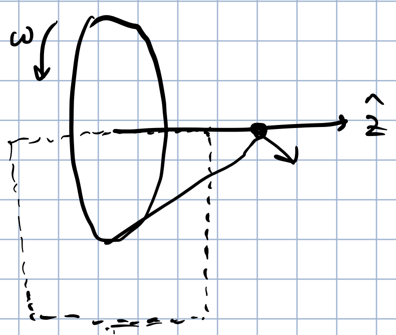
$$d\vec{E}_{\hat{z}} = \frac{\lambda\,dl}{4\pi\varepsilon_0(R^2+z^2)}\cos\theta\,\hat{z}$$

$$= \frac{\lambda\,dl}{4\pi\varepsilon_0(R^2+z^2)}\frac{z}{\sqrt{R^2+z^2}}\hat{z}$$

$$= \frac{z\lambda R\,d\theta}{4\pi\varepsilon_0(R^2+z^2)^{3/2}}\hat{z}$$

$$\vec{E}(z) = \int_0^{2\pi}\frac{z\lambda R\,d\theta}{4\pi\varepsilon_0(R^2+z^2)^{3/2}} = \boxed{\frac{\lambda R z}{2\varepsilon_0(z^2+R^2)^{3/2}}\hat{z}}$$

1b)



$$d\vec{B} = \frac{\mu}{4\pi}(I\,d\vec{l}\times\hat{r})/|\vec{r}|^2$$

orthogonal to each other!

$$= \frac{\mu}{4\pi}(\lambda\omega R\,d\theta\,\hat{z}\cos\theta)/R^2+z^2$$

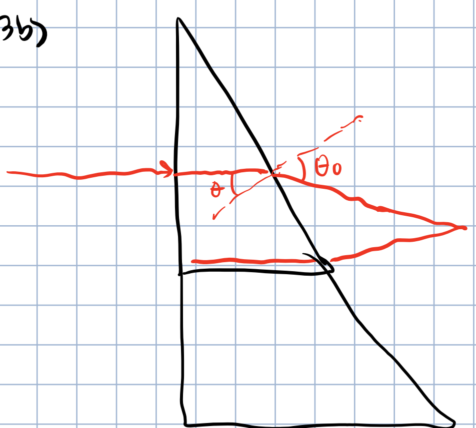
$$\vec{B} = \frac{\mu\lambda\omega R z}{4\pi(R^2+z^2)^{3/2}} * 2\pi\,\hat{z}$$

$$= \boxed{\frac{\mu\lambda\omega R z}{2(R^2+z^2)^{3/2}}\hat{z}}$$

maybe review ampere's Law & amperian loop drawing.

3a) Intensity should match that of the original incident wave.
homogenous & bright

3b)



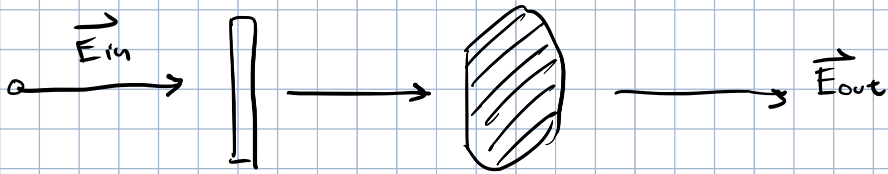
$\Delta\varphi = k\Omega = \frac{2\pi}{\lambda}$

2 a) Raw Power: $\frac{100\,W}{mm^2} * \pi\left(\frac{1}{2\sqrt{\pi}}\right)^2$

$= 100 * \pi\left(\frac{1}{4\pi}\right) = 25\,W$

need to be able to make 0 W.

use $\lambda/4$ plate and linear polarizer in series.



$\lambda/4$
(RCP - Linear)

Linear Polarizer
at an adjustable
angle θ

b) min power: Linear polarizer is at 90° to the Linearly polarized light exiting the $\lambda/4$ plate. $\boxed{0\,W}$

max power: " " 0° " " $\boxed{25\,W}$

c) Brewster's angle occurs where $r_p = 0$!

d) use glass and place at Brewster's angle to incident laser.
use reflected beam which has only ½ intensity as new input to system in 2a).
Power still varied through rotating polarizer ✓

config for 0 power stays same
max power now 12.5 W

Re-do problems!