ME 109 midterm 1

conduction: $q''_x\ [W/m^2] = -k\frac{dT}{dx}$ $\quad k\ [W/m\cdot K]$

convection: $q''\ [W/m^2] = h(T_s - T_\infty)$ $\quad h\ [W/m^2 \cdot K]$

radiation: $q'' = \varepsilon\sigma(T_s^4 - T_{sur}^4)$ $\quad \sigma = 5.67\times10^{-8}\ [W/m^2K^4]$

OR $q\ [W] = h_r A (T_s - T_{sur})$ $\quad \varepsilon = (0,1)$ "emissivity" — perfect reflector, perfect absorber

where $h_r = \varepsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2)$

Fourier's law: $q_x = -kA\frac{dT}{dx}$ (1-D) [W]

$\vec{q}'' = -k\nabla T = -k\left(\frac{\partial T}{\partial x}i + \frac{\partial T}{\partial y}j + \frac{\partial T}{\partial z}k\right)$ [W/m²]

(3-D version, not area normalized)

$$\underbrace{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}_{q''_{in} - q''_{out}} + \underbrace{\dot{q}}_{\dot{E}_{gen}} = \underbrace{\rho c_p\frac{\partial T}{\partial t}}_{\dot{E}_{st}}$$

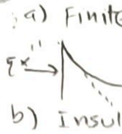
Types of Boundary conditions

① const. surf. Temp. $T(0,t) = T_s \quad \forall t$

② constant heat fluxes.

a) Finite $-k\frac{\partial T}{\partial x}\Big|_{x=0} = q''_x$

b) Insulated (adiabatic) $-k\frac{\partial T}{\partial x}\Big|_{x=0} = 0$!



④ Convection.

$-k\frac{\partial T}{\partial x}\Big|_{x=0} = h[T_\infty - T(0,t)]$

Thermal Resistances

$R_{cond} = \frac{L}{kA}$, $R_{conv} = 1/hA$ } cartesian

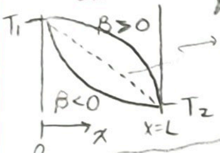
$R_{cond} = \frac{\ln(r_1/r_2)}{2\pi Lk}$, $R_{conv} = 1/2\pi r L h$ } cylindrical

Series: $R = \Sigma R_i \rightarrow \frac{1}{k} = (\Sigma\frac{1}{k_i})^{-1}$ OR $\frac{1}{h} = (\Sigma\frac{1}{h_i})^{-1}$

Parallel: $\frac{1}{R} = (\Sigma\frac{1}{R_i})^{-1} \rightarrow k = \Sigma k_i$ OR $h = \Sigma h_i$

Variable Thermal Conductivity ($k = k_0(1+\beta T)$)

$-k\frac{\partial T}{\partial x} = q''$ (const for GEOMETRICALLY symmetric & no external heat gen $\dot{q}(x)$)



$\rightarrow \beta = 0$ $\quad q'' = -\frac{k_0}{L}\left[(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2)\right]$

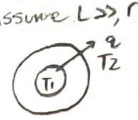
//solved via sep of variables

Spheres & cylinders

$q_r = T_1 - T_2 / \left(\frac{\ln(r_o/r_i)}{2\pi kL}\right)$ [W]

$T(r) = T_1 - \frac{T_1 - T_2}{\ln(r_o/r_i)}\ln(r/r_i)$

Assume $L >> r_i, r_o$

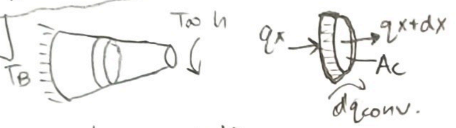


$q = T_1 - T_2 / R_{sph}$ where $R_{sp} = \frac{r_o - r_i}{4 r_o r_i \pi k}$

For solid-sphere w/ heat gen.

$T(r) = \underbrace{-\frac{\dot{q}r^2}{6k} + \frac{\dot{q}r_o}{3h} + \frac{\dot{q}r_o^2}{6k}}_{C} + T_\infty$

Fins



$q_x - q_{x+dx} = dq_{conv} = h(\partial A(x))[T(x) - T_\infty] = h(Pdx)[\Delta T]$

$q_x = kA_c(x)\,dT/dx$

Gov Eq: $\frac{d}{dx}\left(-kA_c(x)\frac{dT}{dx}\right) - hP[T(x) - T_\infty] = 0$

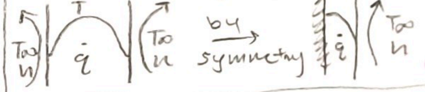
$\Rightarrow \boxed{\frac{d^2T}{dx^2} - \frac{hP}{kA_c}[T(x) - T_\infty] = 0} \Rightarrow \boxed{\frac{d^2\theta}{dx^2} - \frac{hP}{kA_c}\theta = 0}$

//homogenous 2nd order DE!

Aside: $\boxed{\frac{\partial^2\theta}{\partial t^2} - m^2\theta = 0 \Rightarrow C_1e^{mt} + C_2e^{-mt}}$

Key substitutions: $\boxed{m^2 = hP/kA_c}$ $\boxed{\theta = T(x) - T_\infty}$

wall w/ heat gen



Gov Eq: $\frac{\partial^2 T}{\partial x^2} = -\dot{q}/k$

quick maff ⇓ this one EZ

$T(x) = -\dot{q}/k\, x^2 + C_1x + C_2$

Infinite fin example.

$\lim_{x\to\infty} T(x) = C_1e^{mx} + \cancel{C_2e^{-mx}} \rightarrow 0 = \boxed{k_1e^{mx}}$

$\boxed{\theta(x) = \theta_b e^{mx}}$

$\boxed{q_f = \sqrt{hP_k A_c}\,\theta_{bound}} = -kA_c\frac{d\theta}{dx}\Big|_{x=0}$

(fin heat transfer rate)

Fin effectiveness & efficiency

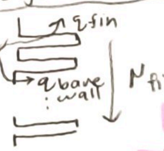
$\boxed{\varepsilon_f = q_f / hA\theta_b} \Rightarrow \frac{\text{fin dissipation}}{\text{bare wall dissip.}}$

Ex:  $P/A = \frac{4}{t}$ ↑ vs $P/A = \frac{20}{t}$ ☺

For infinite fin, $\varepsilon_f = \sqrt{\frac{kP}{hA_c}}$ $\quad$ Note: $\boxed{\varepsilon_f\uparrow \leftrightarrow \frac{P}{A_c}\uparrow}$

$\eta_f = q_f/q_{max} = \boxed{q_f / hA_f\theta_b}$ $\quad A_f$: total Area!

Systems of Fins



$A_{total} = A_{bare wall} + N * A_{fin}$

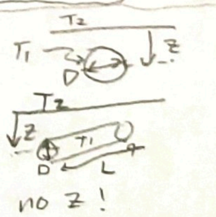
$q_{tot} = \underbrace{N\eta_f A_f\theta_b h}_{\text{fins}} + \underbrace{h\theta_b A_b}_{\text{bare wall}}$ algebra

$q_{tot} = hA_{tot}\left[1 - \frac{NA_f}{A_{tot}}(1 - \eta_f)\right]\theta_b$

$\eta_o = q_{tot}/hA_{tot}\theta_b = \left[1 - \frac{NA_f}{A_{tot}}(1 - \eta_f)\right]$

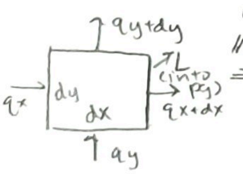
Shape Factors

	Restriction		
① isothermal sphere buried in semi-inf med.	$z > D/2$	$S = \frac{2\pi D}{1 - D/4z}$	
② horz cylinder L buried in semi-inf med.	$L >> D$ & $z > 3D/2$	$S = \frac{2\pi L}{\ln(4z/D)}$	
③ vert cylinder in semi inf med.	$L >> D$	$S = \frac{2\pi L}{\ln(4L/D)}$	no z!
④ Between 2 cylinders in infinite medium	$L >> D_1, D_2$; $L >> w$	S = some shit.	



$q = S\,k\,\Delta T$ $\quad R_{th} = \frac{1}{S k}$ $\quad S = [L]$

2-D Steady Heat Conduction



$q_x + q_y = q_{x+dx} + q_{y+dy}$ //energy conservation.

⇒ taylor expand to first term.

$q_x = -k(dy\,L)\frac{\partial T}{\partial x}$

$q_y = -k(dx\,L)\frac{\partial T}{\partial y}$

Gov Eq: $\frac{\partial}{\partial x}\left(\cancel{k}\frac{\partial T}{\partial x}\right)\cancel{dx\,dy\,L} + \frac{\partial}{\partial y}\left(\cancel{k}\frac{\partial T}{\partial y}\right)\cancel{dy\,dx\,L} = 0$

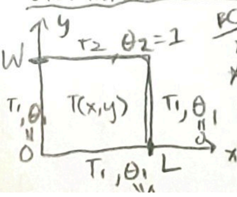
//assuming k is const

$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0} \Rightarrow \boxed{\nabla^2 T = 0}$

Tips for Graphical Transformations:

- $f(x+c)$: shift LEFT c
- $f(x-c)$: shift RIGHT c
- $-f(x)$: x-axis flip
- $f(-x)$: y-axis flip
- $f(y,x)$: x=y line flip

standard basis for superposition



BC

$x=0,\ T=T_1,\ \theta=0$

$x=L\ \ T=T_1,\ \theta=0$

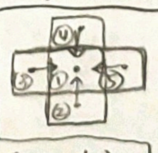
$y=0\ \ T=T_1\ \ \theta=0$

$y=W\ T=T_2,\ \theta=1$

Sub: $\boxed{\theta = \frac{T - T_1}{T_2 - T_1}}$

sol: $\theta(x,y) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1} + 1}{n}\sin\frac{n\pi x}{L}\,\frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)}$

NODAL analysis



$\dot{q}_{①} + \sum_{all\,k} q_{k\to①} = 0$ //energy balance

shortcut: when $\Delta x = \Delta y$ & No heat gen. $T_{m,n} = \frac{T_2 + T_3 + T_4 + T_5}{4}$

Nodal with conv & with composite materials...

Transient Heat Conduction (No Spatial Temp gradient)

@t=0 dip into EXTENSIVE Pool - heat loss via conv.

$E_{st} + E_{out} = 0$

$E_{st} = \rho c \mathcal{V} \frac{dT}{dt}$

$E_{out} = h A_s (T(t) - T_\infty)$

BC: $T(0) = T_i$

Gov. Eq: $\frac{dT}{dt} = -\frac{hA_s(T-T_\infty)}{\rho c \mathcal{V}}$ $T(0)=T_i$

let $\theta = T - T_\infty$ ⇓ solve.

$$T(t) = (T_i - T_\infty) e^{-\frac{hA_s}{\rho c \mathcal{V}} t} = (T_i - T_\infty) e^{-t/\tau}$$

where $\tau = \left(-\frac{hA_s}{\rho c \mathcal{V}}\right)^{-1}$

Lumped Capacitance model applies when $Bi < 0.1$

Critical Dimensional Quantities

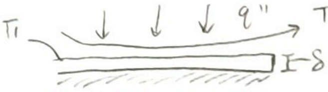
$Bi = \frac{hL_c}{k}$ OR $\frac{h_r L_c}{k}$ for conv/rad resp.

// ratio of conv to conduction capability

$F_o = \alpha t / L_c^2$ // dimensionless time. $\alpha = \frac{k}{\rho c}$ (thermal diffusivity)

$\frac{\mathcal{V}}{A_s} = L_c$ // characteristic length (relevant in Bi, Fo).

Nonhomogenous Lumped Capacitance



Gov Eq: $\frac{dT}{dt} + \frac{h}{\rho c \delta}(T - T_\infty) = \frac{q''}{\rho c \delta}$

solve ⟹ $\theta(t) = \underbrace{\left(-\frac{q''}{h}\right) e^{-\frac{h}{\rho c d} t}}_{\theta gen} + \underbrace{q''/h}_{\theta part}$

Math aside: $\frac{d\theta}{dt} + m\theta = C$

$\Rightarrow \theta(t) = \underbrace{C_1 e^{-mt}}_{gen} + \underbrace{\frac{C}{m}}_{part}$ C_1 determined via BC.

Spatial effects when $Bi > 0.1$.

Dimensionless substitutions:

$\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = f(x^*, F_o, Bi)$

$x^* = x/L$ $t^* = \frac{\alpha t}{L^2} = F_o$

Gov Eq: $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

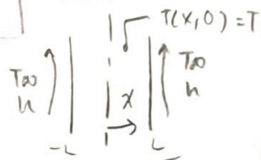
Plane wall w/ convection

$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

$$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 F_o) \cos(\zeta_n x^*)$$

where C_n, ζ_n can be found on table.

// when $F_o > 0.2$, can use a first-term approximation.



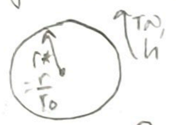
$$\frac{Q}{Q_o} = 1 - \frac{\sin \zeta_1}{\zeta_1} \theta_o^*$$

where $\theta_o^* = \frac{T|_{x=0} - T_\infty}{T_i - T_\infty}$ // temperature @ centerline

ratio of $\frac{\text{total e transferred}}{\text{max e transferrable}}$ ☆

Cylinder w/ convection.

$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$



$$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta^2 F_o) J_o(\zeta_n r^*)$$

NOW, $F_o = \alpha t / r_o^2$ $Bi = \frac{h r_o}{k}$

$$\frac{Q}{Q_o} = 1 - \frac{2\theta_o^*}{\zeta_1} J_1(\zeta_1)$$

Sphere w/ convection

$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

$$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 F_o) \frac{1}{\zeta_n r^*} \sin(\zeta_n r^*)$$

$$Q/Q_o = 1 - \frac{3\theta_o^*}{\zeta_1^3}\left[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)\right]$$

Semi-Infinite Solids

Case 1: Const. Surface BC Temp $T(0,t) = T_s$

$$\frac{T(x,t) - T_s}{T_i - T_s} = \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

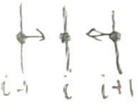
Case 2: Const. Surface heat flux: $q_s'' = q_o''$

$$T(x,t) - T_i = \frac{2q_o''(\alpha t/\pi)^{1/2}}{k} \exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_o'' x}{k} \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Case 3: Surface Convection $-k\frac{\partial T}{\partial x}\Big|_{x=0} = h[T_\infty - T(0,t)]$

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\right]\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

Discrete Transient Numerical Analysis



Gov Eq: $kA\frac{T_{i-1}^p - T_i^p}{\Delta x} - kA\frac{T_i^p - T_{i+1}^p}{\Delta x} = \rho c A \Delta x \frac{T_i^{p+1} - T_i^p}{\Delta t}$

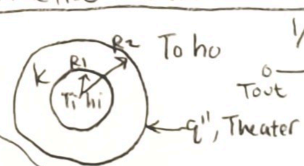
Algebra

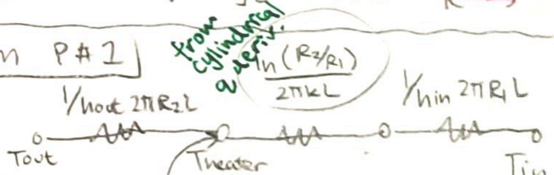
$$T_i^{p+1} = F_{od}\left[T_{i+1}^p + T_{i-1}^p\right] + \underbrace{(1 - 2F_{od})}_{\text{coefficient (of } T_i^p) > 0 \text{ for STABILITY}} T_i^p$$

$F_{od} = \frac{\alpha \Delta t}{\Delta x^2}$

$Bi_d = \frac{h\Delta x}{k}$ discrete versions

Practice Midterm P#1





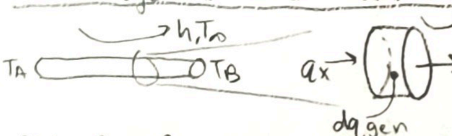
$q'' = q'' 2\pi R_2 L$

Energy balance setup eq:

q_2 Theater q_1 R1 R2 R3 q'' $q'' = q_1 + q_2$

$$q'' = q_1 + q_2 = \frac{T_{heater} - T_o}{R_1} + \frac{T_{heater} - T_i}{R_2 + R_3}$$

Nonhomogenous 1-D Conduction problem



$kA_c\frac{d^2T}{dx^2}dx + \dot{q}A_c dx = hPdx(T - T_\infty)$

$$\Rightarrow \boxed{\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = -\dot{q}/k}$$

Sub $\theta = T - T_\infty$

$\theta(x) = \theta_c + \theta_p$

$\frac{d^2\theta}{dx^2} - \underbrace{\frac{hP}{kA_c}}_{m^2}(\theta) = -\dot{q}/k$

$\boxed{\theta_c = C_1 e^{mx} + C_2 e^{-mx}}$

θ_p = some const

critical def $\cancel{\frac{d^2\theta_p}{dx^2}} \to 0$ $- m\theta_p = -\dot{q}/k \Rightarrow \boxed{\theta_p = \dot{q}/mk}$

$$\boxed{\theta(x) = C_1 e^{mx} + C_2 e^{-mx} + \dot{q}/mk}$$

ME 109 Midterm 2. Cheatsheet.

Boundary Layers

δ = velocity BL δ_T = Thermal BL

Fourier's Law?

Local Surface heat flux: $\boxed{q_s'' = -k_f \frac{\partial T}{\partial y}\Big|_{y=0}}$ → Local heat trans. coeff $\boxed{h = \frac{-k_f \frac{\partial T}{\partial y}\big|_{y=0}}{(T_s - T_\infty)}}$

Newton's Law of Cooling: $\boxed{q_s'' = h(T_s - T_\infty)}$

$\bar{h} \triangleq \frac{1}{L}\int_0^L h_x \, dx \longrightarrow \boxed{q = \bar{h} A_s (T_s - T_\infty)}$ //general heat transfer of ENTIRE plate.

Velocity BL

$C_f = \frac{\tau_s}{\rho u_\infty^2 / 2}$ $\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$

$\delta \triangleq \frac{u}{u_\infty} = 0.99$

Thermal BL

$\boxed{\delta_T \triangleq \frac{T_s - T(\delta_T)}{T_s - T_\infty} = 0.99}$

1 IMPORTANT DIM-LESS Quantities & other relations.

$\boxed{Re_x = \frac{u_\infty x}{\nu}}$ $\boxed{\nu = \mu/\rho}$ $\boxed{\alpha = \frac{k}{\rho c_p}}$

→ internal flow $Re_{cr} = 2300$ ☆
→ external flow $Re_{cr} = 5*10^5$

$\boxed{Pr = \frac{c_p \mu}{k_f} = \frac{\nu}{\alpha} \simeq \frac{\delta}{\delta_T}}$ Relative BL thickness ~ Pr!

$\boxed{Nu_x = \frac{h_x L}{k_f}}$ $\boxed{\overline{Nu}_x = \frac{\bar{h} L}{k_f}}$

ALWAYS TRUE REGARDLESS OF TURBULENCE.

$\boxed{Ra_x = Gr_x Pr = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha}}$

$\boxed{Gr_x = \frac{g\beta(T_s - T_\infty)x^3}{\nu^2}}$ Grashof #

Fluid Mechanics Gov Eq Review

Mass cons.: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Momentum: $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$

Energy: $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$

External Flows

Laminar Flat plate + exact integral method.

Turbulent Flat plate   * discussion printout.

Mixed flat plate

Cylinders

$\boxed{Re_D = \frac{u_\infty D}{\nu}}$ $\boxed{\overline{Nu}_D = \frac{\bar{h}_D D}{k_f}}$

$\boxed{\overline{Nu}_D = C Re_D^m Pr^{1/3}}$ → C & m from T7.2

//most common conditions: $0.4 \le Re_D \le 4 \times 10^5$, $Pr \ge 0.7$

Sphere:

$\boxed{\overline{Nu}_D = 2 + (0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}) Pr^{0.4} * \left(\frac{\mu}{\mu_s}\right)^{1/4}}$

Internal Flows

$\boxed{D_{eff} = \frac{4A_c}{Perim.}}$ //noncircular cylinder

Circular Laminar

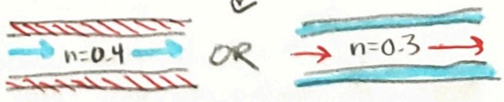
$\boxed{Nu_D = 4.36}$ unif. q_s'' } fullydeveloped
$\boxed{Nu_D = 3.66}$ unif T_s

$\boxed{\overline{Nu}_D = 3.66 + \frac{0.0668 (D/L) Re_D Pr}{1 + 0.04[(D/L) Re_D Pr]^{2/3}}}$ //thermal entry, $Pr \ge 5$

Turbulent.

$\boxed{Nu_D = 0.023 Re_D^{4/5} Pr^n}$ //most common

$Re \ge 10{,}000$
$0.6 \le Pr \le 160$
$L/D \ge 10$



Thermal vs Hydro entry lengths

$\boxed{L_{fd,t} = 0.05 Re_D Pr}$ thermal $\boxed{L_{fd,h} = 0.05 Re_D}$ hydro

Internal Flows Temp. relations

const q_s''

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} \neq f(x) \; !$$

$$\boxed{T_m(x) = T_{mi} + \frac{q_s'' P}{\dot{m} c_p} x}$$

$q_s'' = \text{const}$

const T_s

perimeter

$$\frac{dT_m}{dx} = \frac{P}{\dot{m} c_p} h \Delta T$$

$$\boxed{\frac{T_s - T_m(x)}{T_s - T_{mi}} = \exp\left(-\frac{P x}{\dot{m} c_p} \bar{h}\right)}$$

$$\boxed{\frac{T_s - T_{mo}}{T_s - T_{mi}} = \frac{\Delta T_o}{\Delta T_i}}$$

Additional

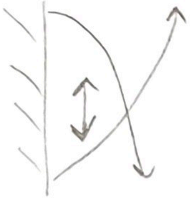
$$T_m = \frac{\int_{A_c} \rho u(r) c_p T(r) \, dA_c}{\dot{m} c_p}$$

// Bulk mean temp.

$$\overline{\Delta T_{am}} = (\Delta T_i + \Delta T_o)/2$$ // mean ΔT

$$\boxed{\overline{\Delta T_{lm}} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)}}$$ // log-mean ΔT

Natural Convection



vertical plates

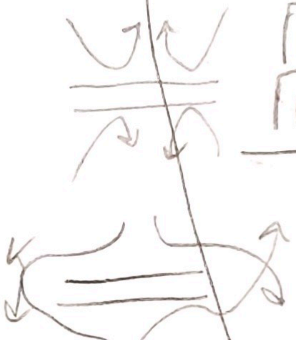
$$\boxed{\overline{Nu_L} = C Ra_L^n}$$

Lam: $C = 0.59$ $n = \frac{1}{4}$

Turb: $C = 0.10$ $n = \frac{1}{3}$

For Laminar: $Ra \leq 10^9$

Horz Plates

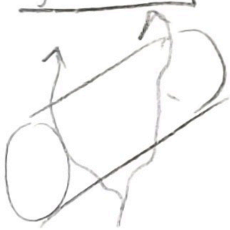


$$\boxed{\overline{Nu_L} = 0.54 Ra_L^{1/4}}$$ $Ra_L \leq 10^7$

$$\boxed{\overline{Nu_L} = 0.15 Ra_L^{1/3}}$$ $Ra_L \geq 10^7$

$$\boxed{\overline{Nu_L} = 0.27 Ra_L^{1/4}}$$

Cylinder



$$\boxed{\overline{Nu_D} = C Ra_D^n}$$

Table 9.1 for C, n

Practice Final

$$\boxed{\frac{T_\infty - T_{mo}}{T_\infty - T_{mi}} = \exp\left(\frac{-UA}{\dot{m} c_p}\right)}$$

Temperature difference in convective tube